APPLICATION OF NON-LINEAR PROGRAMMING FOR OPTIMIZATION OF FACTORS OF PRODUCTION IN MINING INDUSTRY

Tadeusz FRANIK, Ewa FRANIK

Abstract: This paper presents the problem of estimating the optimal quantity of factors of production such as labor and capital in mining and quarrying sector. The optimization criterion is manufacturing cost minimization at given value of production. The value of output produced is described by Cobb-Douglas function and the cost of production by a linear function. Due to the nature of objective function, to solve the problem a non-linear programming method is used after introduction of unmarked Lagrange function multipliers.

Key words: Mining and minerals extraction, factors of production, production function, cost function, non-linear programming.

1. Introduction

One of the main dilemmas in economic governance is problem of selecting appropriate quantitative relations between the factors used in the production process. The most important factors which affect production effects are human labor and capital. The volume of production depends on size of input of these factors, and their proper application are key issues for improving management effectiveness.

With regard to relationship between production output and effort, optimal decision-making occurs in one of two following situations:

– Maximize production output (profits) of at a given level of factor inputs,
– Minimize expenditures at certain level of production output.

Simultaneous optimization of outputs and inputs is a contradictory task. An attempt to determine appropriate relationships between these factors of production for the mining industry in Poland requires determination of functional dependencies between cumulative production outputs and expenditures, and the amount of factors of production involved in the mining process.

The restructuring of the coal mining industry implied significant changes in relationship between factors of production (reduction of employment, reduction of investment, ownership changes, etc.). The volume of production has been considerably reduced which entailed reduction of production capacity.

The problem of measuring the dependencies between inputs of human labor and resources, and equipment of work and the amount (or value) of the resulting product are major problems of the analysis of mining processes. These issues may be addressed with proper application of production function, which represents an econometric model of the relationship between production output and effort. Depending on the degree of aggregation of statistical data used to approximate the structural parameters of the production function, model may include a single mining plant, group of mines or the entire mining sector.

Econometric analysis of the production function, meaning the determination the form of this function on the basis of statistical data, according to Z. Pawlowski [1] is based on the four assumptions that can be formulated as follows:

- Surveyed companies do not change the applied technology and techniques in a random manner, they produce with the same technology over long periods of time;
- Within a specific manufacturing technology, quantitative relationships between the both effort of individual factors and volume of production are either unchanged for long periods of time or occurring changes are slow, regular and systematic;
- Estimation of the production function for group of companies (e.g. industry) is made only when all entities within the group use the same manufacturing technology;
- Econometric production functions describe relationships between inputs and production outputs assuming industry standard manufacturing conditions and average organizational and managerial capabilities. These functions do not describe these relationships obtained under ideal conditions.

The reform of the mining industry, particularly coal mining, is carried out in a planned and systematic way and changes are likely to achieve the planned objectives. In this process level of the production factors changes, and also the productivity of labor and capital factors is different [2]. Since the production function used to describe extraction process in the mining coal sector is a power function, curvilinear programming methods can be used to determine the target relations between labor and capital.

2. The value of production and the cost of production as functions of factors of production

The fundamental problem of the mining plant is to determine the size of the mining extraction using a specific quantity of productive factors, taking into account relationship between prices of these factors and the price that can be obtained from sale of production outputs.

The use of production factors in the economy may be treated as a problem of resources allocation, influenced by three key items:

- The costs of the use of production factors, such as fixed assets, labor, capital, and their prices,
- The production process described by the production function,
- Revenues from sales of production output, driven by market demand and prices.

From the economic point of view, role of the mining plant is to convert purchased productive factors into a product which is then sold in the market. Assuming that main objective of the company is to maximize its profit, it must be estimated by defining costs of production and company's income.

When *r* production factors are used in the mining process, total cost of production, described by the linear function, amounts to [3, 4]:

$$C(\mathbf{x}) = \sum_{i=1}^{r} p_i x_i + K_s = p^T \mathbf{x} + K_s \tag{1}$$

where:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_r \end{bmatrix}$$ – vector of factors of production,

$$\mathbf{p} = \begin{bmatrix} p_1 \\ \cdot \\ \cdot \\ \cdot \\ p_r \end{bmatrix}$$ – vector of factors of production prices,

K_s – intercept.

The production volume described by following relationship:

$$y = f(x) = f(x_{1,\dots,} x_r) \tag{2}$$

Company's revenue from sales of production is:

$$R(\mathbf{y}) = c^T \mathbf{y} \tag{3}$$

where:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}$$ – vector of quantity of produced goods,

$$\mathbf{c} = \begin{bmatrix} q_1 \\ \cdot \\ \cdot \\ \cdot \\ q_n \end{bmatrix}$$ – vector of produced goods' prices.

The company's profit is the difference between revenues and production costs:

$$Z(\mathbf{x}, \mathbf{y}) = R(\mathbf{y}) - C(\mathbf{x}) \tag{4}$$

So if functional dependencies between revenues and costs and expenditures factors of production are known, it is possible to analyze the economic relationship between the effects and expenditures at any of the criteria, such as:

– Maximization of profit with given costs of production,
– Minimization of production costs with given volume of production,
– Limit of the profitability of manufacturing.

Maximizing the Z(x, y) function one can determine volume of production, at which profit is at its maximum or the volume of production as a function of the selling price. As an alternative to the criterion of profit maximization, criterion of minimizing production costs may be used. Then, in the optimization process the C(x) function is being minimized using given size mineral extraction.

3. Approximation of the production function structural parameters and the production costs function

As already mentioned, the production function describes the dependence of production from factors of production. Either production and factors of production may be described using theirs volume and value. Both approaches have their supporters and opponents. When it comes to production, it seems most appropriate to describe it in natural units, especially for such a relatively homogeneous product as, for example, coal. However, when analyzing entire industry, consisting of a number of production units, using natural units is usually impossible. Of necessity, production output is described using its value with restriction that to achieve comparability of the data, proper adjustments reflecting changes in prices of the production outputs must be made.

Likewise, one can calculate human labor needed. When low-level aggregated data available (e.g. mining facility), it can be measured with headcount or work time. When higher level of aggregation is available (e.g. sector, industry), value of human labor seems more appropriate. The most challenging is quantitative description of capital used. In majority of analyzes it is impossible (without using simplifications in describing fixed assets), because of diversity of factors of production. Additional constraint when gathering statistical data regarding this variable, is its actual usage (not book value usage). Another difficulty is connected to lack of information of fixed assets usage in mining process. High value of company's fixed assets does not have to be correlated with their high productivity [5].

When approximating structural parameters of production value function and costs function all variables were considered at their yearly value (both dependent and independent) in mln zł/ year.

Calculations are based on statistical data from years 2002 – 2011, the period of significant changes in mining industry in Poland. Nominal values, calculated at their real values, were converted into prices of the basis year 2011. Capital is described with gross value of fixed assets and human labor is calculated by multiplication of average yearly employment in the mining industry and gross yearly salary. The data were presented in table 1.

Value of production sold in the industry is described with two-factor Cobb-Douglas type power function [8, 9]:

$$Y_t = \beta X_{1t}^{\alpha_1} X_{2t}^{\alpha_2} \zeta_t \tag{5}$$

where:

Y_t – value of production in *t* period,
X_{1t} – human labor in *t* period,
X_{2t} – capital in *t* period,
$\beta, \alpha_1, \alpha_2$ – structural parameters of production function,
ζ_t – error term.

Tab. 1.The source data for estimation of parameters of structural functions of production and function costs, [mln zł]

Years	Value of production	Cost of production	Human Labor	Capital
1	*2*	*3*	*4*	*5*
2002	38 921,2	38 971,3	11 463,2	52 275,9
2003	47 337,0	36 920,5	10 667,3	45 778,2
2004	43 295,2	36 798,4	11 054,5	45 144,8
2005	42 870,7	36 925,8	11 173,4	46 361,1
2006	46 099,3	39 182,0	11 398,3	49 196,3
2007	46 590,9	39 618,4	11 728,6	54 282,5
2008	50 460,6	43 432,1	11 517,9	55 897,1
2009	47 106,6	42 848,2	13 106,9	59 594,6
2010	51 414,1	43 090,9	12 295,2	54 255,3
2011	64 589,0	45 890,0	12 120,1	53 530,0

Source: Own elaboration on the basis of statistical data [6, 7]

Production cost in the industry is described with two-factor linear function:

$$C_t = a_0 + a_1 X_{1t} + a_2 X_{2t} + \zeta_t \qquad (6)$$

where:

C_t – cost of the production in t period,
a_0, a_1, a_2 – structural parameters of cost of production function,

Structural parameters a_1 and a_2 are unit costs (prices) of involved factors of production.

Estimation of structural parameters of examined functions resulted in following values:

- for production function:
  β = 14,5601
  α1 = 0,7447
  α2 = 0,1030
- for cost of production function:
  a0 = 7660,97
  a1 = 0,9995
  a2 = 0,4079

Measure used to assess quality of econometric model's fit to the actual data is coefficient of agreement. It is relation of sum of actual and theoretical data standard deviations square powers and average value of the parameter. Its value should be within the range (0,1) and the lower the value is, the better "quality" of the model.
In analyzed case, coefficients of agreement are:

- for production function 0,3814
- for cost of production function 0,3514

Values of correlation coefficients are acceptable and estimated econometric models of described economic phenomenons may be used for the purpose of analysis of these processes.

Figure 1 presents value of production function in mining sector – actual and approximated with estimated production function, whereas figure 2 presents cost of production – actual and theoretical, respectively, calculated with estimated production cost function for mining sector.

Presented processes visualize increasing trend in value of examined variables in period after restructuring of mining sector.

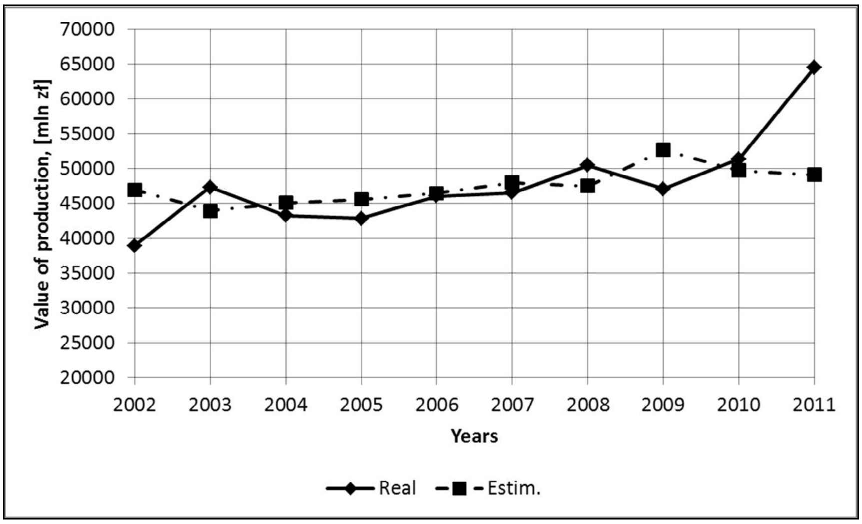

Fig. 1. Actual and estimated production function value (prices at level of 2011)

4. Optimization of the structure of factors of production in mining and extraction sector

The problem of determination the optimal size of each factor of production may be simplified to problem of determining minimum value of cost production function $C(X_1, X_2)$, considering limitation described by production function. Therefore, production costs may be minimized at given (constant) value of production. Since limitation is described with power function, the decision problem may formulated as non-linear program and solve it with Lagrange method of undetermined multipliers.

In this method, if objective function f(x) has unconditional extremum, which exceeds given limitations, then objective functions converts to Lagrange function.

$$L(\mathbf{x},\lambda) = f(\mathbf{x}) + \sum_{i=1}^{r} \lambda_i \, g_i(\mathbf{x}) \tag{7}$$

where λi are undetermined Lagrange multipliters, whereas $g_i(x)$ are functions of *x* vectors, being limitations in mathematic model of decision task. These functions may be linear or non-linear and, with regard to both objective function and limitations, it is assumed that they are continuous.

In this method, seeked conditional extremum of objective function is being replaced with unconditional Lagrange function. To determine optimal values of decisive variables, partial derivatives of Lagrange function with regard to decision variables and Lagrange multipliers, and then compared to zero.

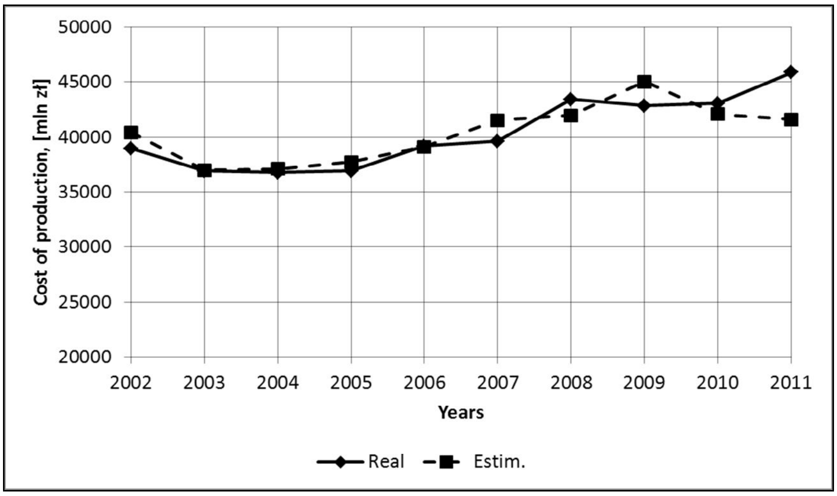

Fig. 2. Actual and estimated cost production function value (level of prices with year 2011)

In this case, the problem may be simplified to determining optimal values of X_1* and X_2* variables, minimizing following objective function:

$$\min C(X_1, X_2) = \min(7660{,}97 + 0{,}9995\,X_1 + 0{,}4079\,X_2) = \\ = C(X_1^*, X_2^*) \quad (8)$$

Considering limitation:

$$Y_0 = 14{,}5601 X_1^{0,7447} X_2^{0,1030} \quad (9)$$

where:

Y_0 – constant value of production.

Additionally, mathematic model is being constrained with bondary conditions regarding non-negative value of decision variables.

Considering present value of production, objective function converted to Lagrange function is:

$$L = 7660{,}97 + 0{,}9995\,X_1 + 0{,}4079\,X_2 + \lambda(64589 - \\ -14{,}5601 X_1^{0,7447}\, X_2^{0,1030}) \to \min \quad (10)$$

This way, seeking for unconditional extremum of production cost function may be replaced with seeking of conditional extremum of Lagrange function.

When constant value of production at a current level equal PLN 64 589 mln (base year prices), optimal factors of production may be determined.

We calculate partial derivatives of Lagrange function with regard to decision variables and Lagrange multiplier and compare them to zero:

$$\frac{\partial L}{\partial X_1} = 0{,}99 - \lambda 10{,}84 X_1^{-0,25} X_2^{0,10} = 0 \tag{11}$$

$$\frac{\partial L}{\partial X_2} = 0{,}41 - \lambda 1{,}50 X_1^{0,75} X_2^{-0,90} = 0 \tag{12}$$

$$\frac{\partial L}{\partial \lambda} = 64589 - 14{,}56 X_1^{0,74} X_2^{0,10} = 0 \tag{13}$$

Sufficient condition for existence of minimum of analyzed function is that the determinant of the matrix of second order partial derivatives is positive and all main minors of the matrix to are positive.

System of equations above may be solved by eliminating multipliers λ from formulas (11) and (12):

$$\lambda = \frac{0{,}091}{X_1^{-0,25} X_2^{0,10}} \tag{14}$$

$$\lambda = \frac{0{,}273}{X_1^{0,75} X_2^{-0,90}} \tag{15}$$

Comparing these formulas results in:

$$X_1 = 3{,}0 X_2 \tag{16}$$

This relation sets optimal relation between human labor and capital (in this case – gross value of fixed assets), enabling achieving production outcome of a given value.

From formula (13), X_1 may be determined:

$$X_1 = \sqrt[0,75]{\frac{4436{,}1}{X_2^{0,10}}} \tag{17}$$

Which after putting into formula (16) enables to determine optimal values of decision variables:

X_1^* = 22 224,69 mln zł,

X_2^* = 7 408,23 mln zł.

The annual cost corresponding to optimal values of decision variables amounts to 32,896 mln zł (all of above values refer to prices at level of 2011).

At the same time it means that, the optimal structure of considered factors of production, it means the ratio of human labor to capital effort for mining sector should amount to 3.

The ratio of capital to the effort of human labor, i.e. labor ratio (capital-labor ratio) in this case is amount to 0,33 zł / zł, when in fact in the considered period it was much higher and it exceed 4 zł / zł.

The capital-labor ratio can be calculated:

$$u = m * w \quad (18)$$

where:

u – capital-labor ratio (refers to the equipment, machinery, vehicles etc. that a business uses to make its product or service),

m – capital intensity,

w – labor productivity.

If we assume that the capital-labor ratio is constant (in fact, it changes slightly), then it can be concluded that labor productivity is a linear function of the capital (technical devices), i.e. the labor productivity growth can be achieved only as a result of increasing the capital-labor ratio. At constant employment and constant labor effort, it requires increasing the capital expenditures to the extent that is required in relation to labor productivity growth. So, if for example we postulate an increase in productivity by 10%, then with optimal structure of labor and capital, it requires a capital increase by 10%, or invest 740.8 mln zł.

It is assumed that in the manufacturing process, to some extent, it is always possible to exchange one factor of production for another, maintaining a constant level of production [10, 11]. Theoretical substitution possibility of a single factor for another can be determined from the production function, converting it to a form, where one factor is a function of the other (at a constant value of output Y_0). Graphically, this represents a function called curve leveled production (isoquant). Moving along the curve leveled production, it can be determined how much of one factor of production can be replaced by a second factor without changing the level of production.

The equation of the isoquant for the Cobb-Douglas production function have the form:

$$X_2 = f(X_1) = \left(\frac{Y_0}{\beta}\right)^{\frac{1}{\alpha_2}} X_1^{-\frac{\alpha_1}{\alpha_2}} \quad (20)$$

Figure 3 shows a sample of curves leveled production for production with a value of 40000, 50000 and 60000 mln zł, determined on the basis of the functions of the estimated structural parameters.

The productivity of analyzed factors of production, defined as the ratio of production value to inputs of a specific factor during the period of time were as follows:

– Productivity of human labor – from 3,59 to 5,39 - average over the period 4,12 zł/zł,
– Productivity of capital – from 0,79 to 1217 - average over the period 0,91 zł/zł.

In the case of the optimal structure of factors of production relevant indicators of productivity are:

– Productivity of human labor – about 2,91 zł/zł,
– Productivity of capital – about 8,41 zł/zł.

It follows that the actual productivity of human labor (average during the period) is close to the theoretical value, while the theoretical productivity of capital (i.e., the gross value of fixed assets), is significantly higher than the corresponding values actually occurring in the mining industry.

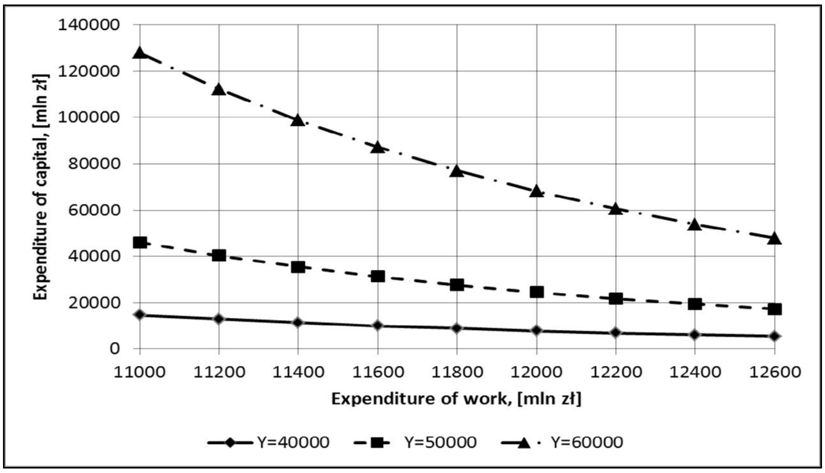

Fig. 3. The theoretical dependence of expenditure of capital from expenditure of work with assumption of constant value of production

5. Conclusions

The analysis of productivity of factors of production across the mining industry in the long term may indicate significant trends in terms of their use and the proper way of managing them. Economic transformations carried out in Poland have a fundamental importance for improving the effectiveness of economic management. The reform of the mining industry plays a particularly important role in this process, due to the current role of the industry in the national economy and the depth and scope of changes. Improving efficiency is always associated with better utilization of the factors of production. Currently, mining companies do not always use sufficiently methods of data analysis to inform about the usage of factors of production. Evaluating the effectiveness of factors of production at the level of individual operators, the most intuitive approaches are most commonly used, however they do not have a scientific justification.

Acquisition of macro-economic analysis of the entire mining industry in a long period of time, finds its justification in the fact that as we move from the quantitative development to qualitative development and in the case of limiting the free productive forces, it increase the degree of capital intensity of production. At the same time it may cause a decrease in productivity of fixed assets, while reducing the human labor and objectified, production volume per unit.

In terms of restructuring the mining industry the economic analysis should be more widely used. Especially in terms of dredging the methods for testing the usage of factors of production in connection with the analysis of production and financial results. It can make a significant contribution in improving the use of reserves inherent in the assets of mining companies, especially in the field of production factors such as production assets.

The methods used in the context of operational research, including non-linear programming can be a useful tool in making rational decisions in the management of production factors, determine the optimal structure and the possibility of their mutual substitution.

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Dr inż. Tadeusz FRANIK
Katedra Ekonomiki i Zarządzania w Przemyśle
AGH Akademia Górniczo-Hutnicza im. St. Staszica
30-059 Kraków al. Mickiewicza 30
tel.: (0-12) 617 21 37
e-mail: franik@agh.edu.pl

Mgr Ewa FRANIK
Alcon Pharmaceuticals Ltd a Novartis Company
Fribourg, Szwajcaria