RISK MANAGEMENT OF THE INVESTMENT PROJECT AT RANDOM CASH FLOWS

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Abstract: In the paper the analytic method of estimation of risk in mining investment projects was introduced. The elementary cash flows were treated as random variables about well-known schedules of probability. The time of exploitation of project was treated similarly at random. The distribution of net present value of an investment can be described by computing its mean and variance. The expected value and variance yield considerable information about the riskiness of the investment. It in method uses the continuous models of cash flows as well as continuous discounting for calculation of net present value.

Key words: mining investment projects, risk estimation, random cash flows, random variable probability distribution functions.

1. Introduction

Mining investment operations are accompanied by relatively large risks. Contrary to operations realised in other economic sectors operations concerning mining are characterised by a significant uncertainty both in the nature state, internal conditions and the environment state. The results of a thorough analysis of the project risk constitute a necessary information source for the investor, since the proper identification of the risk and its management allows for successful planning, designing and realisation of the investment.

A proper identification and quantification of potential hazard fields is an important element of the project prerealisation study. Examples of hazardous areas characteristic for large mining investments are:
- row materials and materials risks,
- risk related to the applied technology,
- enterprise management risk (process supervision, contractors and performance coordination),
- risk related to selecting of subjects present at the investment realisation,
- risk related to the realisation (technological breaks, variable natural conditions and deposits),
- market risk (changes of the economic situation, cyclical changes of the economic situation),
- finance risk (sources of funds, financing structure, payback system),
- credit risk (changes of crediting terms, credit cost, inability to repay a credit),
- other misfortunes.

M. Wilczek [1] considers a project risk management as the art and science of identification, determination and preventing of a risk during the whole project lifetime to be able to effectively realise its aims. The task of risk management is maximising positive effects and minimising negative consequences of all events. One can speak about a risk when the probability of all events, which can occur, is known. The risk - in this aspect - can be determined in various ways. This can be a:
One of the features of projects realised in mining is the necessity of assuming all risks by an investor and bearing the effects. Other attitudes concerning risks, such as their diversification, compensation, transferring on other subjects or avoiding risks are - in this case - of a limited application [2, 3].

2. Methods taking risk into account applied in the estimation process of investment projects

Discount methods are used in estimation processes and analysis of projects. If the net present value (NPV) is used for the economic assessment of the project efficiency the two aspects are universally considered to be essential. The first one is the proper estimation of cash flow sizes generated by the project as well as the time in which they occur. The second one is related to the uncertainty of obtaining cash flows on the estimated level in the future. The project analysis with using the NPV as the criterion solves, to a significant degree, problems of cash flow sizes and time. The problem related to the project risk estimation is not so straightforward - the NPV method does not provide the possibility of direct taking into account the risk related to the project.

Several methods of analysis of projects performed under uncertainty conditions can be found in the subject literature. Traditionally these methods are divided into indirect and direct ways of risk taking into account in the project estimation. These methods can be qualified - from the technique point of view - either as deterministic or as probabilistic approaches to risk analysis in the project estimation.

The risk, in the probabilistic approach, can be defined as a probability of obtaining unfavourable results. This what is considered unfavourable depends, of course, on what is expected in the given moment. Investments are realised with a hope of obtaining the expected favourable result, often in the indefinite future. This situation causes that all investments are risky, however their essence is never avoiding risks but rather finding the way of their management. The investor inclined to take a risk prefers the risky investment to a cautious one, since the most often it gives a chance for better profits. Simultaneously his inclination to take a risk, represents a high probability of changing a profit value in relation to the expected value. Therefore, usually, the measure of the possible deviation from the planned profit is applied as the risk measure [4, 5].

For example, in relation to expenditures borne during the investment realisation phase, one can speak about risk when the borne realisation costs of individual objects of the project $N_i$ will be higher than the planned cost estimate $N_{kz}$.

\[
N_k \leq N_{kz}, \quad \text{lack of risk} \\
N_k > N_{kz}, \quad \text{risk} \quad (1)
\]

If the investor has at his disposal the properly large information number it is possible to prepare the statistical description of the borne realisation costs $N_k$ by means of a random variable.
On this basis the risk can be defined as the probability of exceeding the assumed price level.

\[ R(N_{kr}) = P\{\omega : N_k(\omega) > N_{kr}\} \]  

(3)

As can be seen, the risk is a function of the assumed cost estimate \( N_{kr} \). If we know the distribution function of the random variable \( N_k \),

\[ \phi_{N_k}(N_{kr}) = P\{\omega : N_k(\omega) < N_{kr}\} = \int_{-\infty}^{N_{kr}} f_{N_k}(x) \, dx \]  

(4)

The risk can be described as follows:

\[ R(N_{kr}) = P\{\omega : N_k(\omega) > N_{kr}\} = 1 - P\{\omega : N_k(\omega) \leq N_{kr}\} = 1 - \phi_{N_k}(N_{kr}) \]  

(5)

Historically speaking, one of the first estimation methods of the project estimation - still readily applied by investors - is the method of expenditures refundation period. The investor bases his decisions on the assumption that the shorter period of the investment expenditures refundation the less risky is the project. When such approach is applied to the investment risk analysis it is assumed that the investment assures positive net cash flows during the payback period, but cash flows occurring later are still uncertain and in this method practically not existing. Since this method, in its classical form, does not take into account the time influence on money value changes, does not consider cash flows after the payback period and does not consider uncertainties of cash flows in the future, it should be considered as not suitable for the risk estimation and not satisfying measure for undertaking investment decisions.

Another approach to the risk problem in the project assessment is an application of conventional, deterministic method of the net present value with a discount rate increased by a certain value taking risk into account. The determination of the proper correction for the riskfree discount rate as well as the application of point estimations of cash flows in the future (which are then discounted by means of the discount rate taking risk into account) is the main difficulty of this method. However, this method is considered to be better than the payback period method.

A different approach to the risk analysis represents the certainty equivalent method, in which the discount rate free from risk is used for the cash flows actualisation, while the prognosed cash flows are corrected by means of risk coefficients. Since correcting of flows concerns each subperiod taken into account, the determination of certainty equivalent coefficients is one of the basic flaws of this method.

Another method, from the deterministic methods group, is the sensitivity analysis, in which an effect of the influence of changes of the key parameters of the investment project on the level of changes of the measure used for the estimation of the developing venture, is applied. Usually, the maximal and minimal values of the parameter estimation, which can undergo changes during the project lifetime, are taken into account and in such a way, after performing the project estimation, the extreme values of measures used for the estimation
and variability range of these results, are obtained. The sensitivity analysis is undoubtedly an effective technique supplying a lot of valuable information concerning the project, specially when it is applied together with other methods of risk analysis.

The main drawback of the characterised deterministic methods, is using by them point values of estimations of the future cash flows, which - in addition - are often estimated too optimistically. This concerns very often institutional investors, who want at all costs to substantiate the need (and economic benefit) of the investment realisation. These point values of cash flows estimations do not depict clearly uncertainties, which are - in a natural way - related to the future cash flows. They do not show variability of data characterising the investment project and totally avoid the uncertainty problem.

Methods, which in a special way take into account uncertainties related to the future cash flows, are quite fast developing during the last period. A probability and probability distribution, which using informatics systems provide efficient tools facilitating decision taking due to obtaining much more information, are utilised in these techniques.

The main good point of the application of simulation methods for estimations of the efficiency of investment projects is overcoming of losses occurring in deterministic risk analysis methods. The simulation assures more convenient usage of the probability distribution than point estimates of the investigated economic parameter. This technique is elastic in using the selected quality measures and offers a tremendous tool for analysing the project sensitivity. Possibly the most important merit of this technique is the possibility of realisation the project management process by means of the estimated parameters of the probability distribution.

In spite of the fact, that the simulation provides special merits in the investment risk estimation and becomes the most liked approach to project managements, it has several shortages. Since it is based on experimental methods it provides only statistical estimations instead of explicit results. Obtaining results by means of these methods is usually a complicated process, since the simulation model has to be developed, computer program written and calculations performed.

Another approach, considered in the subject literature [5, 6, 7] as better in the risk estimation, is using the known probability distribution for the description of cash flows generated by the investment project. This is the analytical way based on properties of statistical distributions, used in order to obtain the probability distribution of the coefficient, which is considered to be the measure of the investment efficiency. The most often such measure constitutes the net present value (NPV) or the internal return rate (IRR). It is possible to differentiate, two main cases: net cash flows in successive years of the investment exploitation are independent from each other and these flows are ideally correlated. The second case is much more complex, nevertheless, similarly as in the first one, annual cash flows can be characterised by their expected value and a variance in such a way as to have the probability distribution function of the present value of cash flows at least similar to the normal distribution. In such case it is enough to know only the mean and variance of the present value for the investment project estimation. The risk measure is in this case the variance (in fact: the standard deviation).

This way of the risk estimation takes into account uncertainty of information, treating the future cash flows or values from which they depend as random variables. It allows to avoid the computer generation of information in the laborious simulation process, (but does not liberate from the verification concerning the assumed probability distribution function). The most important aspects related to the risk estimation, with taking into consideration the described above approach, are presented in the hereby paper.
3. Estimation of the investment project when its efficiency measure is a random variable

If the NPV is used for the economic estimation of the project, its value (assessed deterministically) depends directly only on the net cash flows (NCF) and discount rates. The cash flow is then treated as the sequence of discrete cash vectors, concentrated at the end of successive years of the calculation period.

If the problem is considered probabilistically, then for the risk estimation of the realised investment project the probability distribution of the net present value should be determined. Such determination should take into account randomness of cash flows related to the project: investment expenditures flow, net cash flow related to the usage period and residual values flow, corresponding to revenues from sale of redundant capital assets. This cash flow, in large mining investments such as building and mining operations in a new mine, can be related - in practice - to the whole exploitation period and can be considered as the random variable. The period of the investment exploitation, which significantly influences the estimation, can be also treated as the random variable. Randomness of the deposit mining time is related to a certain variability of deposit resources since their size depends on the market economic situation and on geological deposition of strata.

When the continuity of cash flows (which depends from the NPV) and the variability of the project ‘service life’ are taken into account and at the application of continuous discounting, the NPV can be the random variable, which expected value can be determined from the following equation [1, 8]:

\[
E[NPV] = - \int_{0}^{b} \int_{I_1}^{I_2} [I(t) f(I) dI] e^{-\rho t} dt + \int_{N_1}^{N_2} \int_{Z_1}^{Z_2} [Z(t) g(Z) dZ] e^{-\rho t} dt f(n) dn +
\]

\[
+ \int_{N_1}^{N_2} \left[ L(n) h(L) dL \right] e^{-\rho t} f(n) dn
\]

(6)

where:

- \( I(t) \) - capital expenditures flow as a time function. The random variable with the probability distribution function – \( f(I) \), for \( I_1 \leq I \leq I_2 \),
- \( Z(t) \) - refunding of expenditures as a time function. The random variable with the probability distribution function – \( g(Z) \), for \( Z_1 \leq Z \leq Z_2 \),
- \( L(n) \) - residual value of fixed assets in time \( t = n \). The random variable with the probability distribution function – \( L(n) \), for \( L_1 \leq L \leq L_2 \),
- \( n \) - project duration time treated as the random variable with the probability distribution function – \( f(n) \), for \( N_1 \leq n \leq N_2 \),
- \( b \) - investment realisation time,
- \( \rho \) - intensity of continuous interest, \( \rho = \ln(1 + r) \),
- \( r \) - discount rate devoid of risk.

The above equation can be presented as the sum, which takes into account the expected values of the discussed cash flows.
This equation for calculating the net present value consists of three essential parts: investment expenditures flow, net cash flows from the period of the investment usage and capital assets residual value. If distributions of these cash flows are independent the NPV variance can be expressed by the following dependence:

\[
E[\text{NPV}] = - \int_0^b E[I(t)] e^{-\rho t} \, dt + \int_{N_1}^b E[Z(t)] e^{-\rho t} \, dt \, f(n) \, dn + \int_{N_1} E[L(n)] e^{-\rho t} \, f(n) \, dn
\]  

(7)

\[
V[\text{NPV}] = \int_0^b V[I(t)] e^{-2\rho t} \, dt + \int_{N_1}^b \int_{N_1}^n V[Z(t)] e^{-\rho t} \, dt + V[L(n)] e^{-2\rho n} \, f(n) \, dn
\]  

(8)

where \(V[I(t)]\), \(V[Z(t)]\), and \(V[L(n)]\) denote the variance of investment expenditures flows, net cash flows from the investment exploitation period and capital assets residual value - respectively.

The condition for utilising these dependencies for the estimation of risks related to the investment being assessed is the description of cash flows generated by the investments by means of known distributions (i.e. distributions of known parameters) or by their combinations.

4. Example of the application of the presented risk estimation procedure for the investment project estimation

The example of the application of the presented risk estimation method concerns the management of the part of the deposit in the existing mine. Elementary cash flows, treated as random variables are described by means of the following probability distributions [6,9]:
- investment expenditures - by the exponential distribution,
- flows during the deposit mining period - by the monomodal distribution,
- residual value - by the exponential distribution.

The expected NPV was determined as the measure of the investment project efficiency and the variance together with the corresponding standard deviation NPV - as the risk measure - was calculated for the determined parameters of the given above distributions and for the values on which cash flows depend.

As a partial example of the determined analyses parameters influence on the risk level, its dependence on the operation time of the discussed deposit part and on the discount rate level, was presented.

It was already mentioned that it was difficult to estimate explicitly the deposit mining time in mining investment projects. The dependence of the risk (standard deviation NPV in thousand zl) on the prolongation of the project exploitation time, is shown in Fig. 1. As can be seen in the diagram the prolongation of this period from 2 to 20 years, (above the initially assumed time), causes the risk increase by more than 73 %.

The rate applied for the cash flows actualisation is another value - also characterised by uncertainties - on which the project efficiency significantly depends. At a continuous
actualisation a discrete discount rate corresponds to the continuous interest intensity, however it is still the constant parameter of the account of the investment efficiency. Both, the efficiency and risk of the investment project are immanently related to the level of this rate.

The graphical presentation of the dependence (used as a percentage) of the relative change of the expected NPV (efficiency) and standard deviation (risk) on the continuous interest intensity applied for the cash flows actualisation is shown in Fig. 2. As it is well known, in the majority of classical investment projects (contrary to the so-called ‘credited’ projects) the project efficiency decreases when the discount rate increases. This dependence is also seen in the analysed project. However, it occurs that, the increase of the discount rate influences the NPV standard deviation much more than the decrease of the expected value.

![Fig. 1. Risk dependence on the increased exploitation time of the investment project](image.png)
This means that projects estimated by means of the relatively high discount rate are characterised by the out-of-proportion lower risk level than it results from the project efficiency decrease. The dependence of the risk measure on the efficiency measure, of the investment project analysed as an example, is presented in Fig. 3 (all values in thousand zl).
The relationship presented in the diagram is of the exponential character and can be described by the equation:

\[ y = 14,748 \ e^{0.0009 \ x} \]  

(9)

where:
\[ y \] - standard deviation (risk measure),
\[ x \] - expected NPV (efficiency measure).

This relationship was approximated for variable interest intensity from the range: 2 to 20 %. Its correlation coefficient is very high and equals 0.997.

5. Conclusions

The risk management is one of the fundamental problems related to the investment process. There are various risk sources, however it can be stated that their basic reason is the lack of complete information concerning the future cash flows generated by the project.

The problem of the estimation of the risk related to the investment project, when cash flows are treated as random variables is presented in the hereby paper. The base of the estimation of the project efficiency constitutes the expected value of the determined efficiency measure (e.g. the net present value), while the base of the risk estimation is the standard deviation. If the net present value has the distribution similar to the normal distribution then applying the accumulated distribution function makes it possible to determine the probability with which the applied efficiency measure will take the assumed value. This result will facilitate decision taking by the investor, characterised by the determined approach to risks.

The presented method of the calculation of the expected value and the NPV variance of the investment project, in contrast to the simulation methods, is based on utilising known probability distributions and on estimating their parameters for the description of the elementary cash flows generated by the project.

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References


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