THE USE OF HUNGARIAN METHOD IN THE EVALUATION OF PRODUCTION EFFICIENCY

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Summary: Productive activity of companies is dependent on internal as well as external conditions of their functioning. Customers' requirements concerning products are growing and are being individualised all the time. Such situation makes the rules of acting on the market become more strict. Fast technological development leads to creating new technologies and shortens the life of products. Spending resources, power supplies and other components necessary in a production process causes not only economic but also organisational threats. Presented factors have a major impact on the efficiency of production process. In this manuscript attempts to present the use of Hungarian method in relation to production efficiency in organisational aspect.

Key words: production efficiency, Hungarian method.

1. Introduction

The issue of the production efficiency, despite many efforts and attempts, has not been so far comprehensively presented and developed. At present, companies operating in the dynamically changing environment, take decisions, which have very often the negative influence on the efficiency of individual processes. The main reasons of such a situation are unclear procedures for the evaluation of the efficiency. According to the Authors, the issue of the evaluation of the production process is the very important component of the effective business management, which requires the deeper analysis. [1, p. 137]

The problem of production efficiency does not only apply to companies but also supply chains in which a given company is a link. Efficiency is a key factor which should have influence on the whole supply chain integration on the operational as well as on the strategic level [2, p. 617-626]. Lack of supply chain integration can cause the threat of 'wandering' efficiency issue. Therefore, production efficiency should be analysed in two areas: efficiency of a supply chain and efficiency of a company and its production process. A chance for coherence of the aims of production efficiency in both areas is adhering to the assumptions of Balanced Scorecard, which has been shown in Co-author's publication [3, p. 93-102].

It needs to be considered that an increase in one department's efficiency does not have to result in an increase in whole company's efficiency. Only an increase in key processes efficiency will result in an increasing the indicators of efficiency of a company's business activity. A very important aspect is also coordination of operational and strategic aims. If operational aims do not reflect accurately strategic aims, then a result can be generating contradictory indicators which have a negative influence on production management efficiency. [3, p. 101]

2. Model for assessing production efficiency

Production efficiency is a concept which is quite difficult to define. Generally efficiency can be defined as a measurement (usually expressed as a percentage) of the actual output to the standard output expected. Efficiency measures how well something is performing relative to existing standards; in contrast, productivity measures output relative to a specific input, e.g., tons/labor hour. Efficiency is the ratio of [4]:

- actual units produced to the standard rate of production expected in a time period,
- standard hours produced to actual hours worked (taking longer means less efficiency),
- actual volume of output in value to a standard volume in a time period in value.

The problem of production efficiency is based not only on economic aspects and the standards for setting and assessing these aspects. The possibilities of assessing and analysing economic efficiency are presented by the co-author in his publication [3, p. 93-102]. The present chapter aims at presenting the issue of using mathematical methods for setting operating efficiency of a production process in its organisational aspect. In order to achieve it, the model for assessing production efficiency requires generating credible, detailed and up-to-date information and input data. On the basis of the literature research as well as observations in business practice of manufacturing companies, it should be noted that the basic data, necessary to create the multivariate model of the evaluation of the efficiency of the production process, should be generated from the three main ranges [1, p. 137]:

- the selection of a suitable production management concept,
- balancing of production resources and production capacities,
- the analysis of the performance of the evaluated production process.

Deliberately structured analytical ranges can be seen as an algorithm of the preliminary analysis of the efficiency, which refers to three levels of the management: strategical, tactical and operational. For this reason, the comprehensive analysis of the evaluation of the efficiency should be supported by the preliminary researches conducted at all decision-making levels in the enterprise. The algorithm of the processes of the acquirement of input data for the model of the evaluation of the production efficiency, has been shown in Co-author's publication [1, p. 137-150].

Operating efficiency is a ratio (represented as a percentage) of the actual output of a piece of equipment, department, or plant as compared to the planned or standard output [4]. The effectiveness of assessing operating efficiency depends on the skill of transposing strategic aims to tactical and operating levels. Effective execution of strategic plan needs its translating into action, task results and indicators of everyday activities. The success on the market is attainable by communicating strategic and operating goals on the each level of organisational structure and their connecting with budget of units or employee motivation [5, p. 45]. The transporting idea is based on the assumption of expenses optimisation as a more efficient tool of improving company's result. Expenses optimisation is about rationalising factors which can be steered by a company and for this reason it has a tremendous effect on the possibility to generate higher profits. However, it needs to be remembered that optimum concentration on the analysis of production expenses is advisable only in a situation when the value of logistic expenses rate, meaning the share of logistic expenses in company's total expenses, is significant [6, p. 312]. For this reason operating efficiency deals with the issue of optimisation and rationalisation of a production process in its organisational and technological aspects. Figure 1 shows the place of operating efficiency in the model for assessing production efficiency.

<table>
<tr><td colspan="3">Production efficiency</td></tr>
<tr><td colspan="2">Economic efficiency</td><td>Operating efficiency</td></tr>
<tr><td>Efficiency
of the Organization</td><td>Efficiency
of the Process</td><td>Efficiency of the Job
(Work Station)</td></tr>
</table>

Fig. 1. The place of operating efficiency in the model for assessing production efficiency

The present collation is based on efficiency division presented by G. Rummler, A. Brache [7, p. 31-77] and including the distinction between economic efficiency and operating efficiency. Economic efficiency, according to Strategic Scorecard assumptions, is about analysing efficiency from financial perspective, customer's perspective, internal processes perspective and development perspective. It can be concluded, from figure 1, that operating efficiency is determined by the following cases subjected to controlling analyses:

– when the number of work posts (L_{SR}) is much more bigger than the number of production operations (L_{OP}),
– when the number of work posts (L_{SR}) equals the number of production operations (L_{OP}),
– when the number of work posts (L_{SR}) is much smaller than the number of production operations (L_{OP}).

On the basis of these considerations one can try to create algorithms of optimisation actions with the use of particular mathematical methods. The choice of particular mathematical methods is supported by a detailed analysis of subject matter literature [8; 9], as well as by observations of phenomena taking place in economic practice. As the issue of operating efficiency is very complex, the present manuscript concentrates on discussing the algorithm of dealing with a situation when the number of work posts equals the number of production operations. The algorithm also takes into account indirect options, the situations when:

– the number of work posts (L_{SR}) is much more bigger than the number of production operations (L_{OP}),
– the number of work posts (L_{SR}) is much more smaller than the number of production operations (L_{OP}).

Suggestions for using mathematical method in individual options are presented according to algorithm for assessing operating efficiency in organisational aspect.

The analysis of the three options is the basis for suggested optimising actions as it makes the decisionmaker carry out an analysis of the most efficient task allocation, an alternative use of outsourcing to carry out certain production operations and keeping optional excess of production capabilities in case of posts underload taking into consideration environmental factors at the same time. The authors believe that the chosen mathematical method fully reflects the peculiarity of a production process. In the following chapter the authors present an analysis of cases based on theoretical considerations included in publication [10, p. 19-67, 135-157].

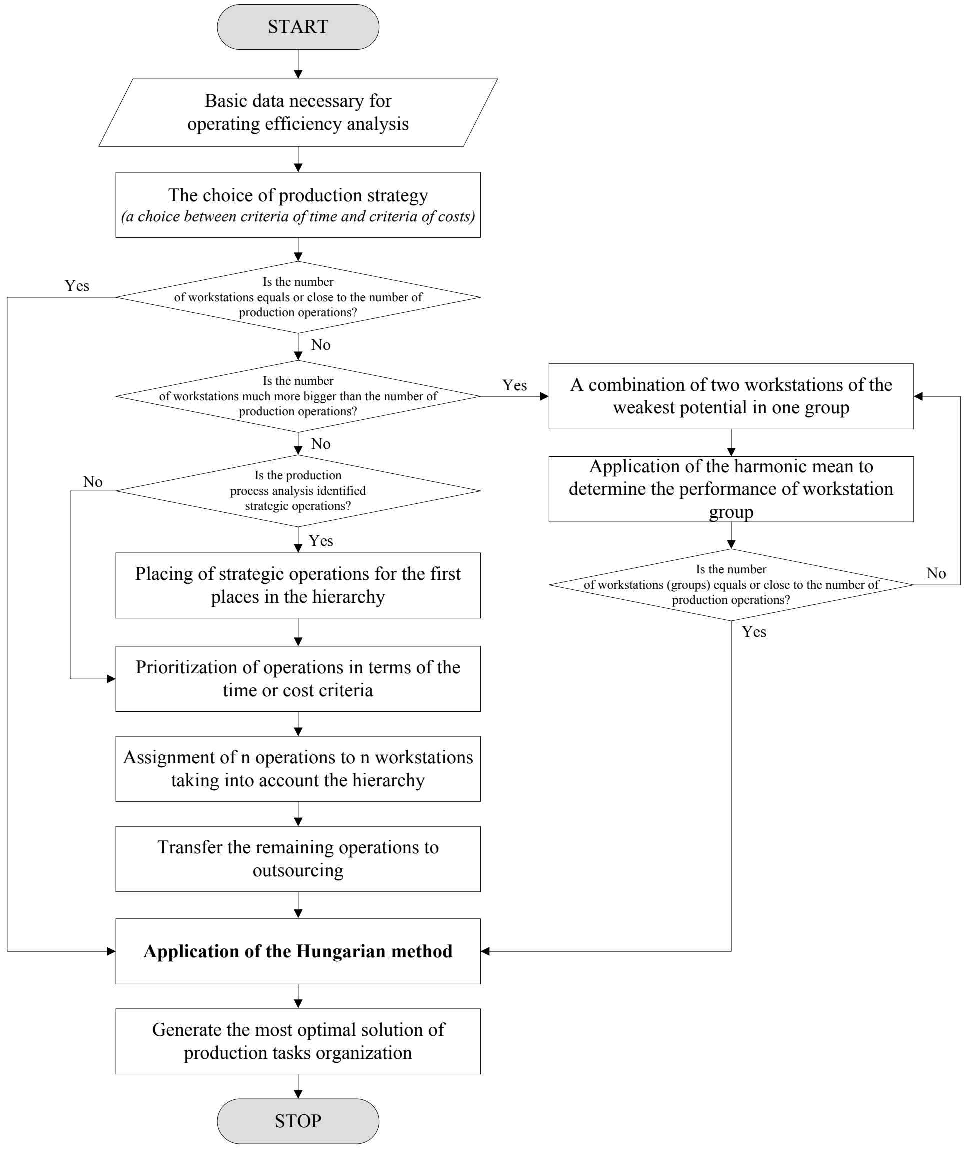


Fig. 2. Algorithm for assessing operating efficiency in organisational aspect

3. Case studies

Presented algorithm includes the process of analysis and assessment of operating efficiency with the use of the Hungarian Method.

The authors chose Hungarian Method as it helps to solve the problem of allocation concerning the same number of production operations as well as work posts realising these tasks. Thus, it can be assumed that there is the possibility of filling *N* of work posts with *N* of production operations.

We know the effects of i – this work post for j – this task (production operation). These effects can be assessed positively (efficiency, the value of production for the time unit) or negatively (the number of shortages, the time of work, production expenses). The effects, therefore, can be described by the matrix $A = [a_{ij}]$, where a_{ij} means the effect obtained by i – this work post for j – this task (production operation).

3.1. The number of work posts equals the number of production operations

Let us consider the following example: In a given production company there are three work posts: Machine A, Machine B and Machine C, each of them is adjusted to carry out production operations: Task 1, Task 2 and Task 3 (these tasks form a production process with the time, in a form of matrix, for carrying out the given tasks. The main aim of operating efficiency analysis is setting optimum allocation of tasks to work posts so that the time of carrying out these tasks can be as short as possible. Table 1 presents unit times of individual operations carried out by given work stations.

Tab. 1. Unit times of operations carried out by machines

Work stations	Production operation		
	Task 1	Task 2	Task 3
Machine A	8	10	7
Machine B	9	11	9
Machine C	10	7	8

Solving this problem of allocation with the help of Hungarian Method means carrying out reasoning based on a few steps.

STEP 1

The matrix of ratios of aim function $A = [a_{ij}]$ should be transformed in a way to make at least one zero appear in each column.

$$A = \begin{bmatrix} 8 & 10 & 7 \\ 9 & 11 & 9 \\ 10 & 7 & 8 \end{bmatrix}$$

The smallest of values should be defined in each column and it should be subtracted from each value in the column. After that we obtain the matrix:

$$A = \begin{bmatrix} 0 & 3 & 0 \\ 1 & 4 & 2 \\ 2 & 0 & 1 \end{bmatrix}$$

It should be noticed that there is no zero in the second verse. Thus, the smallest value in this verse is one and this should be subtracted from the remaining values of the verse.

$$A = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

STEP 2

In the transformed matrix of ratios of the aim function the verses and columns containing zeros should be crossed out with possibly the smallest number of lines (horizontal and vertical). If the number of lines necessary to cross out all zeros equals the size of the matrix *N,* we can set an optimum solution and then move on to Step 3. In a reverse situation, when this number is smaller than the subsize of the matrix, *N,* one should move on to Step 4.

$$A = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

If the smallest number of lines equals the size of the matrix (the size of the matrix dim $A = 3$), then, according to the rule described in Step 2, we should move on to Step 3 where an optimum solution is set.

STEP 3

The optimum solution should be set so that “1” values can be found only in those places where zeros occur in the transformed matrix of ratios of aim function (only one “1” can occur in each verse and column).

$$\begin{bmatrix} 0 & 3 & 0 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

Thus, in the analysed case there is just one possibility which complies with the aforementioned conditions.

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

It means that optimum solution for the analysed production process is setting task 3 for machine A, task 1 for machine B and task 2 for machine C. All the tasks will be carried out in the total time of: 7 + 9 + 7 = 23 [min].

In the case where the number of lines in Step 2 would be smaller than 3, one should move on to Step 4.

STEP 4

If the number of lines covering zeros is smaller than the size of matrix (*N*), we should choose the smallest crossed out element in the obtained matrix. This element we should:

- subtract from the elements which are not crossed out,
- add to the elements which were crossed out twice.

The elements crossed out with one line should be left untouched.

In order to solve the problem of maximisation one should transform the matrix of ratios of aim function so that its elements would have the opposite meaning. It can be done by, for example, subtracting from the biggest element all the others elements.

In a situation when the number of production tasks is not the same as the number of work posts carrying out these tasks, a fictional column or verse, which have elements $a_{ij} = 0$, should be added. Then, there are two cases:

- the number of production operations is marginally smaller than the number of work posts carrying out these operations,
- the number of production operations is bigger than the number of work posts carrying out these operations.

3.2. The number of production operations is smaller than the number of work stations carrying out these operations

Let us consider the following example: In a company there are four work posts: Machine A, Machine B, Machine C and Machine D. Each of them can carry out production operations: task 1, task 2 or task 3 (the tasks form a series of a production process) according to matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

A fictional production task should be introduced and the matrix of ratios of aim function takes on the form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix}$$

The time data for carrying out individual operations by the work posts are presented in a form of a matrix and is shown in table 2. The basic task of the analysis and assessment of operating efficiency is setting an optimum allocation of production tasks to work posts in a way ensuring the shortest time of carrying out these tasks.

Tab. 2. Time data of carrying out individual operations by work stations

Work stations	Production operation		
	Task 1	Task 2	Task 3
Machine A	8	10	7
Machine B	9	11	9
Machine C	10	7	8
Machine D	7	6	8

The solution takes on the form:

$$A = \begin{bmatrix} 8 & 10 & 7 \\ 9 & 11 & 9 \\ 10 & 7 & 8 \\ 7 & 6 & 8 \end{bmatrix}$$

According to the assumptions presented earlier, we should create square matrix by adding a fictional task with values which equal zero for each work post. In this situation we obtain:

$$\begin{bmatrix} 8 & 10 & 7 & 0 \\ 9 & 11 & 9 & 0 \\ 10 & 7 & 8 & 0 \\ 7 & 6 & 8 & 0 \end{bmatrix}$$

After transforming the matrix in the first step we will obtain the following matrix:

$$\begin{bmatrix} 1 & 4 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Each verse and column already contain at least one zero one should carry out some cross outs checking if an optimum solution can be achieved:

$$\begin{bmatrix} 1 & 4 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Due to these actions three lines have been obtained, so it is smaller number than the size of the matrix. Thus, the actions described in Step 4 should be taken. The smallest element is 1 and it should be subtracted from all the elements which are not crossed out and added to the elements which are crossed out twice. Then we obtain:

$$\begin{bmatrix} 1 & 4 & 0 & 1 \\ 1 & 4 & 1 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Next we move on to crossing out:

$$\begin{bmatrix} 1 & 4 & 0 & 1 \\ 1 & 4 & 1 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

In the present situation there is the possibility to set an optimum solution:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

After operating efficiency analysis we obtained two optimum solutions:

$$\begin{bmatrix} 8 & 10 & 7 \\ 9 & 11 & 9 \\ 10 & 7 & 8 \\ 7 & 6 & 8 \end{bmatrix}$$

Therefore task 1 can be set to machine D, task 2 to machine C, task 3 to machine A while Machine B will not be used.

3.3. The number of production operations is bigger than the number of work stations carrying out these operations

Parallel situation is when the number of tasks to carry out outnumbers those who carry out these tasks. Then we hire an additional producer.

In such a case we should hire a person who would carry out the task but not have an influence on the optimum solution. Most often they are given the most difficult task, one that is the most time consuming or requires the most expenses.

Let us analyse the following example: In a company there are three work posts: Machine A, Machine B and Machine C which can carry out all of the production operations: Task 1, Task 2, Task 3 or Task 4 (presented tasks form a series of a production process). The effects of carrying out these tasks are presented by the matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

Then the matrix of ratios of aim function takes on the form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The main aim of operating efficiency analysis is delegating work posts to carry out tasks 1, 2, 3 and 4. In order to carry out the tasks one of the production operations should be carried out by the cooperator. The profit from carrying out this production task is not included in the company's profits. The matrix of production process profit (in dollars) from carrying out the production operations by work posts A, B and C is known and it is presented in table 3.

Tab. 3. Financial data of carrying out individual tasks by work stations

Work stations	Production operations profit			
	Task 1	Task 2	Task 3	Task 4
Machine A	8	10	8	17
Machine B	15	12	6	8
Machine C	6	5	10	10

Thus, in this case we deal with the problem of maximisation and subcontracting one of the tasks. The first stage should include forming a matrix:

$$\begin{bmatrix} 8 & 10 & 8 & 17 \\ 15 & 12 & 6 & 8 \\ 6 & 5 & 10 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix should be transformed into a form which allows using the abovementioned algorithm. It can be obtained by multiplying each verse of the matrix by -1 or transforming each element of the matrix by subtracting from the biggest element of the matrix all the other elements.

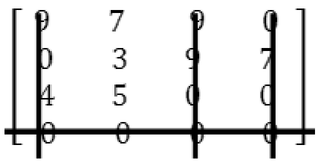
The transformation should by done by subtracting each element of the matrix from the biggest element *($a_{14} = 17$)* thus obtaining the matrix:

$$\begin{bmatrix} 9 & 7 & 9 & 0 \\ 2 & 5 & 11 & 9 \\ 11 & 12 & 7 & 7 \\ 17 & 17 & 17 & 17 \end{bmatrix}$$

Taking parallel steps as in earlier cases we should make the matrix take on the following form:

$$\begin{bmatrix} 9 & 7 & 9 & 0 \\ 0 & 3 & 9 & 7 \\ 4 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We do the cross outs with the smallest possible number of lines:



Optimum solution has been achieved:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 8 & 10 & 8 & 17 \\ 15 & 12 & 6 & 8 \\ 6 & 5 & 10 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

It means that Machine A will carry out task 4, Machine B- task 1 while Machine C- task 3. Task 2 will be carried out by the cooperator who was given this task.

The profit of the production process will be: 17+ 15 + 10 = 42 dollars.

4. Conclusions and further research

The present manuscript discussed the issue of assessing production efficiency with the use of mathematical methods on an operating level only in a situation when the number of work posts equals or is close to the number of production operations. This issue can be implemented with alternative options, for example when the number of work posts is marginally bigger than the number of production operations (one or two posts more). Harmonic average can be used in order to combine the weakest work posts into a group of posts. This method also takes into account the efficiency of individual posts belonging to a group which has a direct influence on each choice of the post with the best potential. Therefore, using arithmetical average is unreal factor.

A detailed analysis should also be carried out for the extreme cases which were only mentioned in this manuscript: when the difference between the number of work posts and production operations is substantial. According to authors' observations, optimising these options can influence organisational aspects of management only in a positive way. This is the direction in which authors' research study is heading. The research study aims at supplementing the algorithm of actions presented in this manuscript and making it more detailed.

It needs to be borne in mind that the model for assessing operating efficiency cannot function irrespectively of economic efficiency aspects. It can lead to contradictory decisions and economic effects. There is the possibility that organisational optimisation of a production process connected with decrease of production portions, frequent change of produced goods or frequent rejig processes has a negative influence on economic efficiency because of increase in shortages and, as a consequence, increase in production expenses. It is a multi-aspect analysis of production efficiency, including operating and economic efficiency aspects, that gives the opportunity for improving the efficiency of a company or supply chain. The issue of production efficiency, in spite of frequent occurrences in literature, is still an issue requiring research and development. The literature cited in the present manuscript is ancient [11, p. 83-97; 10, p. 32-38], which reveals that the methods are already well-known but not used in a present economic practice [13, p. 641-651]. The authors believe that not using proven mathematical models negatively influences controlling actions efficiency which aim at improving production process efficiency. The solutions presented in this chapter should be treated as a suggestion for solving the problems of operating efficiency of a production process in an organisational aspect as this is the aspect that was subjected to authors' theoretical and practical research. Employing mathematical methods makes it possible to avoid mistakes at the stage of achieved results interpretation as well as at the stage of making optimisation decisions. The direction for further study should be minimising the interferences of the input data for the model as well as theoretical assumptions connected with production process organisation.

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